# Significance of M2 and E3 transitions for $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$ configuration metastable level lifetimes

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## Abstract

Magnetic quadrupole and electric octupole transitions from the configurations $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$ were calculated along with magnetic dipole, electric dipole and electric quadrupole radiative transitions in quasirelativistic Hartree-Fock approximation. Their significance in determining the metastable level radiative lifetimes was investigated along several isoelectronic sequences for the ions from $Z = 50$ to $Z = 92$. Strontium-like ions, zirconium-like ions, molybdenum-like ions and rhodium-like ions were studied comprehensively. Remaining isoelectronic sequences with the ground configuration $4d^N$ $(N = 1, 3, 5, 7, 8, 10)$ were also reviewed albeit in less detail. A systematic trends of determined total radiative lifetimes were studied. The importance of magnetic quadrupole and electric octupole transitions from metastable levels of ions from these isoelectronic sequences was investigated and discussed. Inclusion of such transitions of higher multipole order can change theoretical radiative lifetime values for some levels more than ten times. These cases can not be established in advance, without 'performing detailed calculations.

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## I. INTRODUCTION

Theoretical investigation of spectral parameters for multicharged tungsten ions with an open 4d shell [1, 2] has clearly demonstrated that the electric octupole (E3) and magnetic quadrupole (M2) transitions from some levels of the excited configurations $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$ to the ground configuration $4p^6 4d^N$ play very important role in determining their lifetimes. This is caused by the fact that these particular levels are metastable ones with high values of total angular momentum $J$. Hence the electric dipole (E1) transitions from these levels to the ground configuration levels are not allowed by selection rules for $J$. Furthermore, in case when the magnetic dipole (M1) or electric quadrupole (E2) transitions from these levels are weak, their radiative lifetimes $\tau$ are strongly influenced by M2 and E3 transitions to the ground configuration levels.

Transitions of higher multipole order, such as the M2 and E3 radiative transitions, previously were considered in [3–6]. Extensive investigation for the elements ranging from Na-like to Ar-like was presented in [4] where the M2 transitions were computed for magnesium, sulfur and argon isoelectronic sequences. The calculation of Ar-like ions was extended to include E3 transitions. It is easy to explain since there are only few levels with quite high $J$. Various radiative transitions in Ni-like ions were thoroughly studied in [5], including magnetic octupole transitions. Although this paper had presented a huge number of transitions, the cases when the M2 and E3 transitions were significant had not been underlined.

The higher multipole-order radiative transitions were also calculated for $4f^{13}nl$ levels in [6], but their influence on the calculated radiative lifetimes was not revealed. In general, the mentioned works were not specifically dedicated to the investigation of higher multipole-order transitions and did not involve comprehensive study of these transitions. Furthermore, it should be noted, that some experimental studies [7–9] have indicated possible contribution of the E2, M1, E3 and M2 transitions to the metastable level radiative lifetimes $\tau$.

In present work we investigate the influence of M2 and E3 transitions on the calculated radiative lifetimes of metastable levels for a wide range of ions, starting with $Z = 50$ and going up to $Z = 92$ in isoelectronic sequences with the ground configuration $4d^N$. We present comprehensive study for four isoelectronic sequences with $N = 2, 4, 6, 9$. The calculations for remaining open-$d$ shell sequences are performed also, but the results are discussed only in brief.

As a measure of significance of the radiative E2 and M3 transitions, we introduce parameter

$$R = \tau_{\mathrm{E2+M1}}/\tau_{\mathrm{TOT}}, \tag{1}$$

where $\tau_{\mathrm{TOT}}$ is a total radiative lifetime of a level determined from all (E2, M1, E3, M2) transition probabilities (E1 transition is forbidden for these levels), $\tau_{\mathrm{E2+M1}}$ is a radiative lifetime determined from transitions occurring inside the excited configurations $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$ complex. The ratio $R$ shows how much a theoretical radiative lifetime decreases, when M2 and E3 transitions are included to $\tau$ calculation. Here we must underline that all possible transitions with their probability values $A \leq 10^{-12}$ from the levels under considerations are included while determining $\tau$ values.

The radiative transition calculations were performed in a quasirelativistic approximation [10, 11]. The correlation corrections were not included in our calculations, because the main purpose of current work was to determine what kind of transitions are imperative in determining radiative lifetimes of excited levels rather than to calculate very accurate and reliable parameters of radiative transitions. Therefore the ground configuration $4p^6 4d^N$ was investigated in a one-configuration approximation. Furthermore, only the interaction between two excited configurations $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$, which is very strong in multicharged ions, was included for the odd-parity states.

## II. CALCULATION METHOD

We use a quasirelativistic Hartree-Fock approximation (QRHF) in our *ab initio* calculations of ion energy levels and radiative transition parameters, such as transition wavelengths $\lambda$, line strengths $S$, oscillator strengths $f$, transition probabilities $A$. In this approach, the one-electron radial orbitals $P(nl|r)$ are obtained by solving the quasirelativistic equations having the following form:

$$\left\{\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}-V(nl|r)-\varepsilon_{nl}\right\}P(nl|r)-$$
$$X(nl|r)+\frac{\alpha^2}{4}\left(\varepsilon_{nl}+V\left(nl|r\right)\right)^2P(nl|r)+$$
$$\frac{\alpha^2}{4}\left(\varepsilon_{nl}+V(nl|r)\right)X(nl|r)+$$
$$\frac{\alpha^2}{4}\left(1-\frac{\alpha^2}{4}\left(\varepsilon_{nl}+V(nl|r)\right)\right)^{-1}D(nl|r)P(nl|r)=0. \quad (2)$$

The first two lines of this equation represents the traditional Hartree-Fock equations, where $X(nl|r)$ denotes the exchange part of the potential and $V(nl|r)$ represents the direct part of the potential including the interaction of an electron with nucleus $U(r)$ and with other electrons. We take into account the finite size of a nucleus within the nuclear potential $U(r)$ [12, 13]. This allows us to express the radial orbitals in powers of a radial variable in the nucleus region. Next two terms with the multiplier $(\epsilon_{nl}+V(nl|r))$ describe the relativistic correction of the mass-velocity dependence. The last term of equation represents the potential of the electron contact interaction with nucleus.

In our approach we include the contact interaction with the nucleus not only for the $s$ electrons but also some part of that interaction for the $p$ electrons [11]:

$$D(nl|r)=\left(\delta(l,0)+\frac{1}{3}\delta(l,1)\right)\frac{dU(r)}{dr}$$
$$\left(\frac{d}{dr}-\frac{1}{r}\left(\alpha^2Z^2\delta(l,1)\left(-\frac{37}{30}-\frac{5}{9n}+\frac{2}{3n^2}\right)+1\right)\right). \quad (3)$$

A detailed discussion of the particular features of main equation is given in [10, 11], whereas their solution techniques are described in [13, 14]. Concluding the description of the employed approximation, we want to emphasize that our quasirelativistic Hartree-Fock method significantly differs from widely used approach described in [15]. The main differences arise from our adopted set of quasirelativistic Hartree-Fock equations (QRHF) featuring several distinctive properties which are described in more detail elsewhere [10, 11, 13, 14].

The methods to calculate the energy level spectra were discussed extensively in [16]. For the energy level spectra calculation, we include all two-electron interactions in the same way as it is done in conventional Breit-Pauli approximation. This similarity makes it possible to

apply widely used code MCHF BREIT [17] for angular integration of Breit-Pauli Hamiltonian matrix elements. We adopt computer program MCHF MLTPOL [18] to determine the matrix elements of transition operators along with the code MCHF LSJTR [19], which has been adopted for use with the quasirelativistic radial orbitals.

## III. RESULTS

One can define 10 isoelectronic sequences with the $4d^N$ shell. In present work we have investigated isoelectronic sequences with two ($N = 2$, strontium-like ions), four ($N = 4$, zirconium-like ions), six ($N = 6$, molybdenum-like ions) and nine ($N = 9$, rhodium-like ions) electrons in the $4d$ shell of the ground configuration very thoroughly. Remaining isoelectronic sequences with open $4d^N$ shell were examined also. Since the properties (and the behavior of the significance parameter $R$) are very similar, we do not present a detailed analysis of isoelectronic $4d^N$ ($N = 1, 3, 5, 7, 8, 10$) sequences in current work.

We have included ions with comparatively high $Z$ values in order to avoid strong interaction between $4d$ and $5s$ shell electrons. Such an interaction is more significant for elements in low-level ionization degrees. It is obvious that one needs to perform complex calculations with correlation corrections included by adopting extensive configuration bases in order to determine reliable data for radiative transitions or radiative lifetimes. We did not perform this kind of calculations in present study because our main purpose was to examine how inclusion of higher-multipole order transitions changes values of radiative lifetimes $\tau_{\rm TOT}$. It was not an objective of current work to determine high-accuracy $\tau$ values, therefore we did not include correlation corrections in our study. Moreover, we have to point out that the influence of correlation corrections along isoelectronic sequence becomes less significant with increase of $Z$.

For all isoelectronic sequences studied in current work, the levels of excited configurations, which have radiative lifetimes affected by M2 and E3 transitions to the ground configuration, retain the same relative positions in energy level spectra. Nevertheless, their identification, based on the maximum contribution from a particular $LS$ term, can be different. Therefore, this identification in $LS$ coupling scheme is performed for the low-level ionization ion with $Z = 56$.

### A. Sr-like ions

Figure 1 demonstrates energy levels of investigated configurations for the strontium-like ion with $Z = 56$. The excited configuration levels with high total angular momentum $J$ values are presented in a separate column, to the left from the column which shows all energy levels of two excited odd-parity configurations. Hence one can see the location and the quantity of these metastable levels in complete energy level spectra. The E1 transitions are not allowed from these levels. As a rule, the M1 and E2 transitions are allowed from these levels to the lower levels of excited configurations, and M2 and E3radiative transitions to the ground configuration are allowed too. In general case, the M1 transition probability values are the largest ones, whereas the E3 transition probabilities are the weakest ones. Therefore, there exists assumption that the M1 transitions become the most important ones in radiative lifetimes $\tau_{\rm TOT}$ calculation, when E1 transitions are not allowed. In present work, we will investigate properties of such levels further and will demonstrate that this is not a correct estimate for all cases.

There are six energy levels with $J = 6$ and one level with $J = 7$ ($J$-values are given beside corresponding levels) in Fig 1. The ground configuration $4p^6 4d^2$ has only levels with $J = 0, 1, 2, 3, 4$. The transitions, which are significant for the radiative lifetimes $\tau_{\rm TOT}$ and are discussed in Sect. III A also have been presented in Fig 1.

For the ions from the strontium isoelectronic sequence, we analyze the radiative lifetimes of three levels. The lifetimes of one of these levels, namely $4p^6 4d4f\ ^3H_6$, are mostly affected by the M2 transition, although E3 transitions are possible here too. The lifetimes of the second level with $J = 7$ are affected by the E3 transition, because M2 transitions are not allowed from this level. The third level is more particular one, because its lifetimes are affected by both the M2 and E3 transitions. The transition $4p^5 4d^3\,(^4F)\ ^5G_6 - 4p^6 4d^2\ ^3F_4$, presented in Fig. 1 by short-dashed line, demonstrates a particular case, when the excited level can decay not only by M1 transition, but by two additional different-type radiative transitions which have similar transition rate $A$ values.

Here and for other isoelectronic sequences, we present only those transitions, which are most significant for determined radiative lifetimes. Nevertheless, we must underline that all available transitions were included into calculation of radiative lifetimes $\tau_{\rm TOT}$. All possible E2 and M1 transitions were included in $\tau_{\rm E2+M1}$ calculation, too. However, one must keep

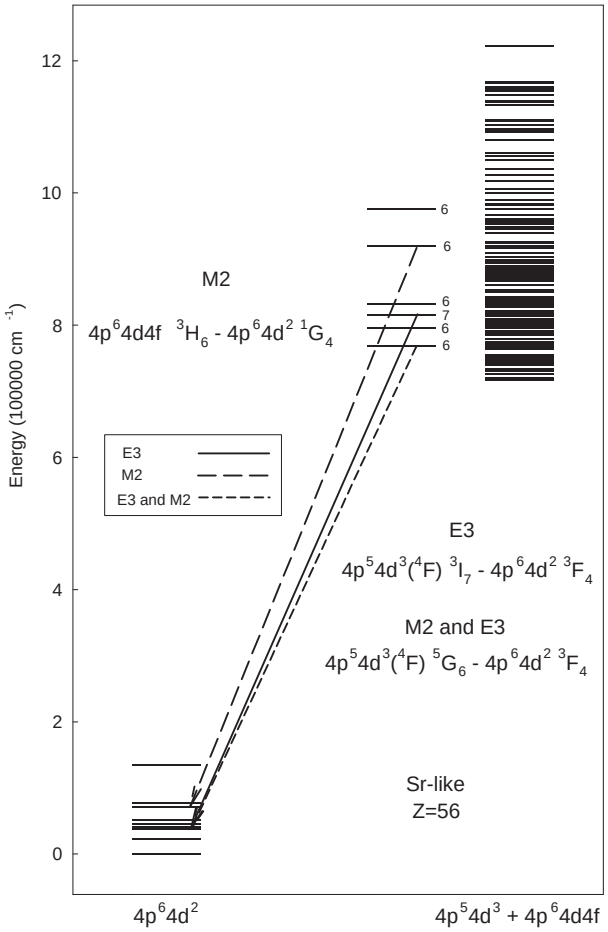


FIG. 1: Energy levels, E3 and M2 transitions for strontium-like barium ion

in mind that the E2 transition probabilities $A$ usually are much weaker than those of M1 transitions. Therefore, the allowed and probable decay channel is attributed to the M1 transitions. In this and further sections, when we refer to allowed (probable) transitions, we mean only the most probable transitions - two or more with the largest transition probability values.

Dependence of the parameter $R = \tau_{\rm E2+M1}/\tau_{\rm TOT}$ on a nuclear charge $Z$ for the level $4p^6 4d4f\ ^3H_6$ in the strontium isoelectronic sequence is presented in Fig. 2, and for the level $4p^6 4d4f\ ^3I_7$, it is given in Fig. 3. The parameter $R$ is significantly larger than 1 at the beginning of isoelectronic sequence, but it decreases fast as the nuclear charge $Z$ increases for these levels as well as for some other levels not presented in plots. In the first case, for the level with $J = 6$, one or two M2 transitions are the most significant. For the next level with

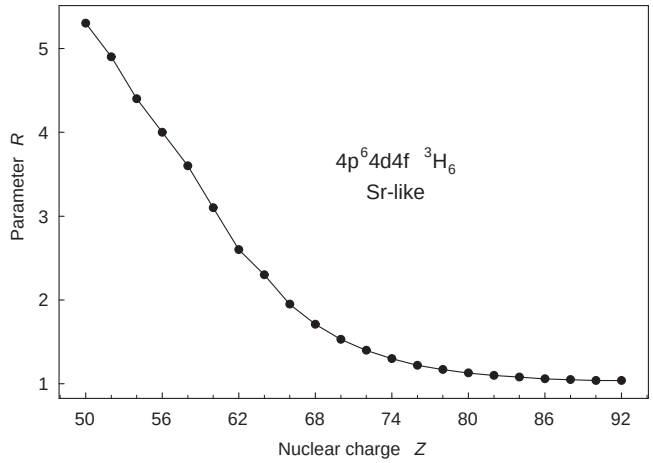


FIG. 2: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^6 4d4f\ ^3H_6$ of strontium-like ions

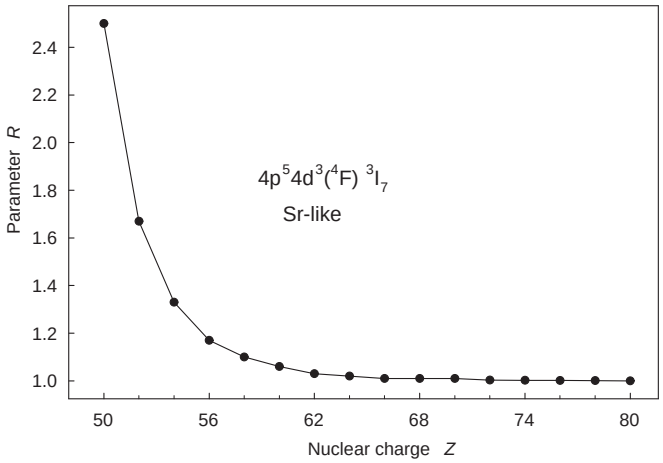


FIG. 3: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^3\ (^4F)\ ^3I_7$ of strontium-like ions

$J = 7$, this behavior is caused by E3 transition. The M2 transitions are not allowed from this levels because the maximum value of total angular momentum $J = 4$ for the ground $4d^2$ shell. We have cut the parameter $R$ dependence in Fig. 3 at $Z = 80$ because it is very close to 1 for higher $Z$, meaning that the E3 transitions are not significant any more.

Completely different dependence of the parameter $R$ on nuclear charge $Z$ is presented in Fig. 4 for the level $4p^5 4d^3\,(^4F)\ ^5G_6$ of the ions from the same isoelectronic sequence. Several decay channels: magnetic dipole transition M1 and two electric octupole transitions

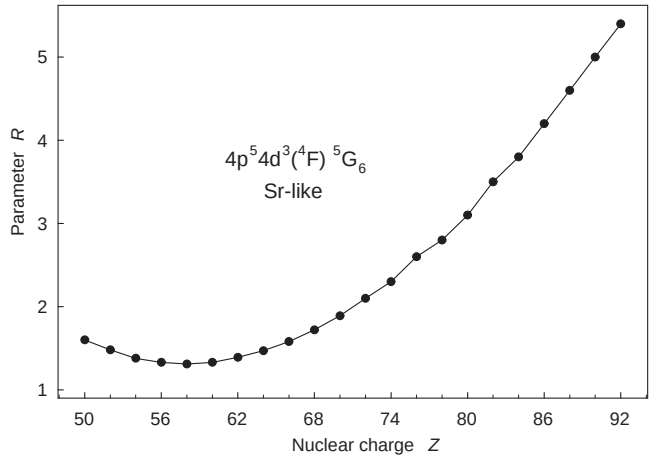


FIG. 4: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^3\ (^4F)\ ^5G_6$ of strontium-like ions

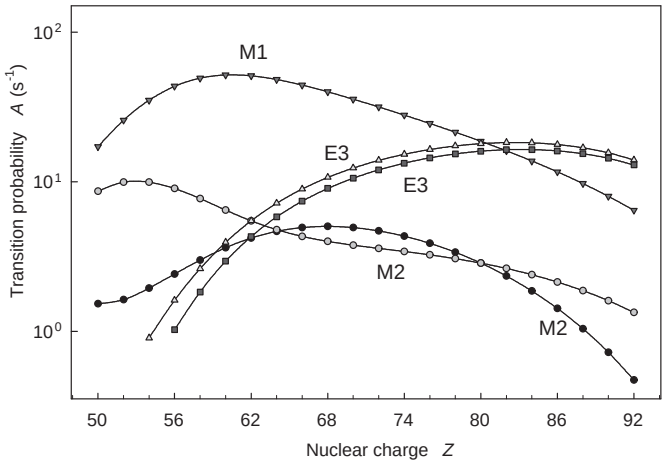


FIG. 5: Radiative transition probabilities originating from the level $4p^5 4d^3\ (^4F)\ ^5G_6$ along the strontium isoelectronic sequence. M1 transitions are presented by grey triangles, two M2 transitions are presented by grey and black circles, E3 transitions are presented by white triangles and black squares.

E3 as well as two magnetic quadrupole transitions M2, are important for this level. The dependences of the mentioned radiative transition probabilities on nuclear charge $Z$ are presented in Fig. 5. Since the probability values change very significantly, a logarithmic scale is used for them here and in later plots. We do not provide identification for the final levels of these transitions deliberately, because the identification of them as well of the upper

level can change along sequence due to a strong mixing of $LS$ terms.

At the beginning of the sequence, where the main decay channel is M1 transition, the parameter $R$ decreases. For higher $Z$ values, the M1 transition probability as well as the M2 transition probabilities decrease whereas the E3 transition probabilities increase very rapidly and, when $Z > 80$, these transition probabilities become most prominent and are most significant for the calculated radiative lifetimes $\tau_{\rm TOT}$.

We want to add that there are other metastable levels with the parameter $R$ values and dependence on nuclear charge $Z$ similar to that presented in Fig. 2.

### B. Zr-like ions

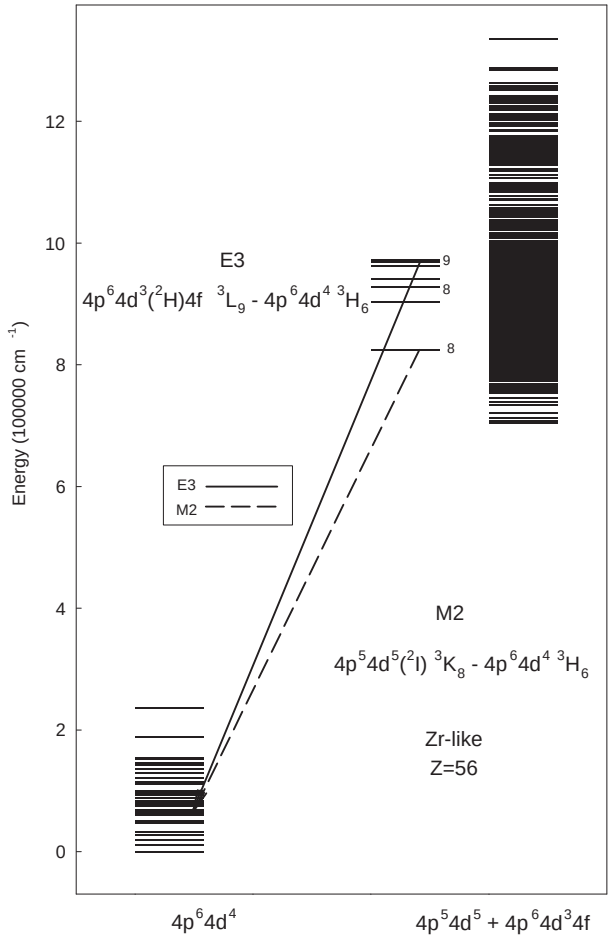


FIG. 6: Energy levels, E3 and M2 transitions for zirconium-like barium ion

Figure 6 presents energy levels of investigated configurations for the zirconium-like ion

with $Z = 56$. The excited configuration levels with high total angular momentum $J$ values are presented in a separate column, to the left from the column which gives all energy levels of the odd-parity configurations like in Fig. 1. The E1 transitions are not allowed from these levels. For the Zr-like ions, we have investigated six energy levels with $J = 8$ and one level with $J = 9$ originating from the complex of configurations $4p^5 4d^5 + 4p^6 4d^3 4f$. The ground configuration $4p^6 4d^4$ for these ions can have the fine-structure levels with $J = 0, 1, 2, 3, 4, 5, 6$. The transitions from metastable levels are presented in Fig. 6 and are discussed further in this section.

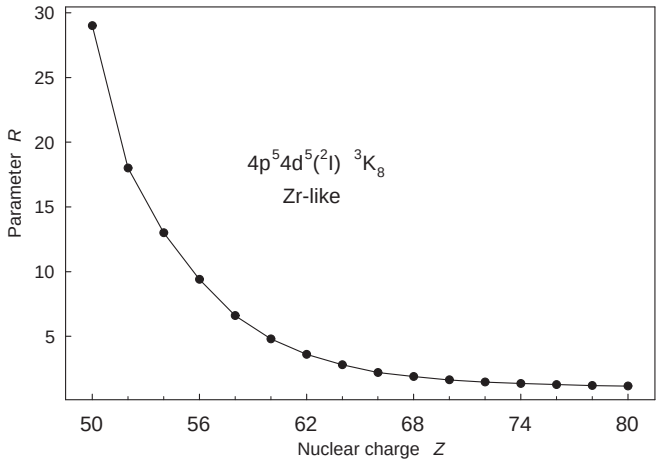


FIG. 7: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^5\ (^2I)\ ^3K_8$ of zirconium-like ions

Figures 7 and 8 show typical behavior of the parameter $R$ for the levels of ions from zirconium isoelectronic sequence. Figure 7 demonstrates the influence of the M2 transition on the determined radiative lifetime $\tau_{\rm TOT}$ for the level $4p^5 4d^5\ (^2I)\ ^3K_8$. That influence is quite formidable at the beginning of the isoelectronic sequence. For other levels with $J = 8$, the parameter $R$ is similar in its value and behavior as in the case of the $4p^5 4d^5\ (^2I)\ ^3K_8$ level, even if these levels belong to another excited configuration, namely $4p^6 4d^3 4f$. We have dropped data for $Z > 80$ from Figs. 7 and 8, because the parameter $R$ is monotonous and very close to 1.

The E3 transitions affect radiative lifetime $\tau_{\rm TOT}$ of the level $4p^6 4d^3\ (^2H) 4f\ ^3L_9$ not so significantly, as it can be seen from Fig. 8. Nevertheless, their influence can not be neglected for small $Z$ values. It should be mentioned that, for the zirconium isoelectronic sequence

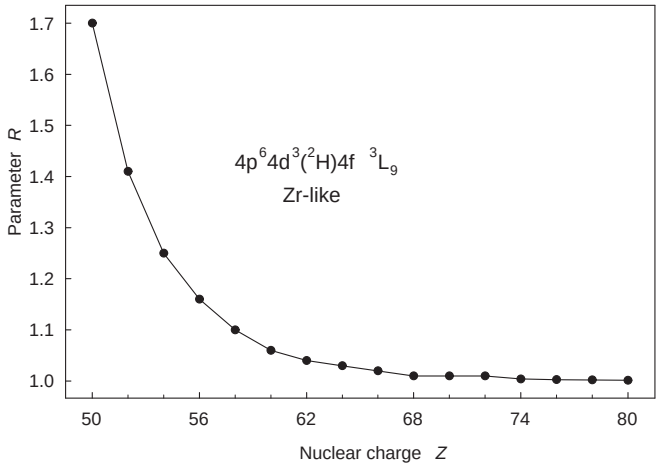


FIG. 8: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^3\,(^2H)4f\ ^3L_9$ of zirconium-like ions

and for all other isoelectronic sequences investigated in current work, the radiative lifetimes $\tau_{\rm TOT}$ for the levels with available M2 transitions have larger $R$ values compared to the levels with only E3 transitions available (see Fig. 2 and Fig. 3). Nevertheless, even if contribution of the E3 transitions to the calculated radiative lifetimes $\tau_{\rm TOT}$ is not so substantial, these transitions must be included into consideration, especially for the levels with the highest $J$ values when the M2 transitions are forbidden.

### C. Mo-like ions

Figure 9 demonstrates energy levels of investigated configurations for the molybdenum-like ion with $Z = 56$. The excited configuration levels with high total momentum $J$ values are presented in a separate column, to the left from the column which gives all energy levels of the odd-parity configurations (like in Fig. 1). There are no E1 transitions from these levels.

The excited configurations $4p^5 4d^7$ and $4p^6 4d^5 4f$ of Mo-like ions have even larger number of metastable levels which have no decay channels through the E1 transitions. There are twelve levels with $J = 8$ and five levels with $J = 9$, see Fig. 9. Moreover, there exists one level with $J = 10$. Therefore, not only the E1 transitions are forbidden from this level. The M2 and E3 transitions also are forbidden, consequently, this metastable level is not related

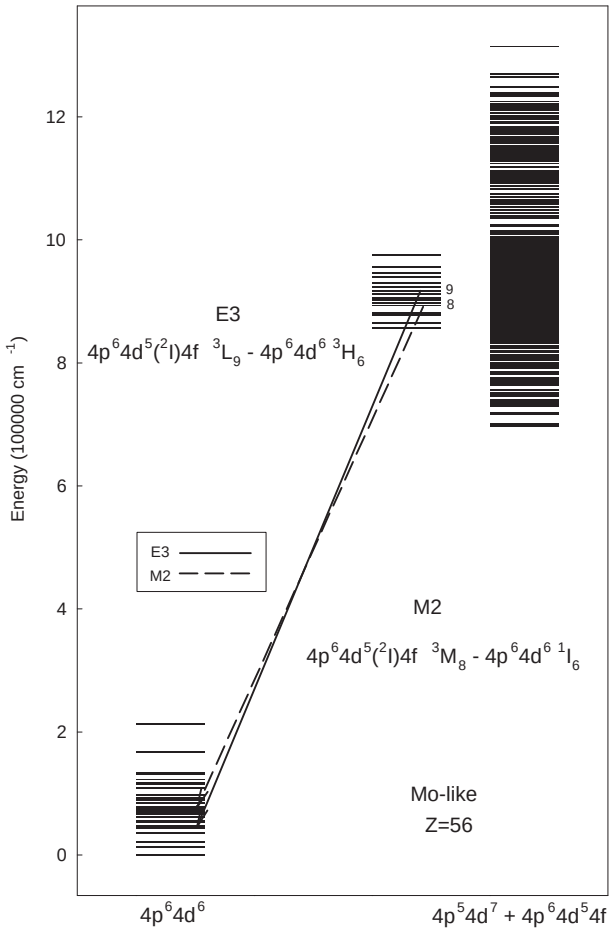


FIG. 9: Energy levels, E3 and M2 transitions for molybdenum-like barium ion

to our study in general. However, it constitutes quite a special case. The lifetime $\tau_{\mathrm{TOT}}$ of this $4d^5\,(2I)\,4f\ {}^3M_{10}$ level is approximately equal to 0.05 s for $Z = 56$ and it decreases to $2.5 \cdot 10^{-6}$ s for $Z = 92$. This radiative lifetime $\tau_{\mathrm{TOT}}$ is defined by decay via two magnetic dipole transitions. The $J$ values for the ground configuration $4p^6 4d^6$ of this isoelectronic sequence is in range from 0 to 6.

In the case of molybdenum isoelectronic sequence, we present the parameter $R$ for two levels. Figure 10 shows the dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^6 4d^5\,(^2I)4f\ {}^3L_9$. Beside the M1 transitions, only E3 transitions are allowed from the level $4p^6 4d^5\,(^2I)4f\ {}^3L_9$ to the ground configuration. Although the investigated configuration $4p^6 4d^5 4f$ has five levels with the total angular momentum $J = 9$, the mixing of terms is not significant here. Consequently, the parameter $R$ decreases very consistently for this

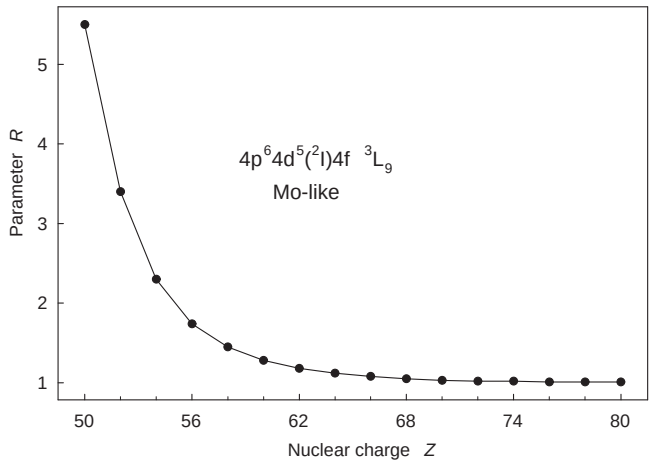


FIG. 10: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^5\,(^2I)4f\ ^3L_9$ of molybdenum-like ions

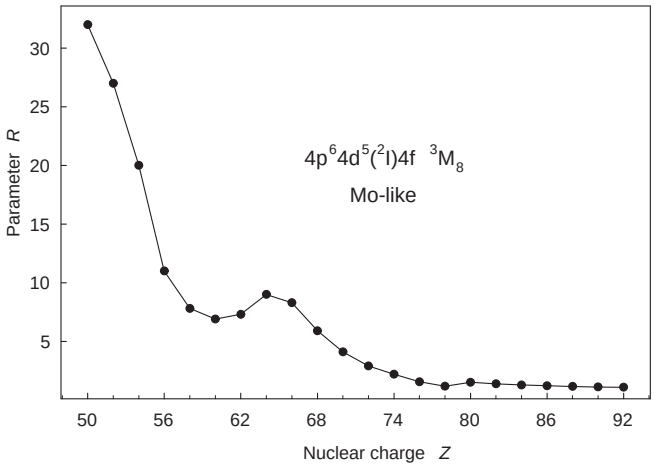


FIG. 11: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^5\,(^2I)4f\ ^3M_8$ of molybdenum-like ions

configuration.

A completely different behavior of $R$ for the level $4p^6 4d^5\,(^2I)4f\ ^3K_8$ is presented in Fig. 11. Radiative transition probabilities $A$ for this level, presented in Fig. 12, clarify such an unusual behavior of the parameter $R$ along the isoelectronic sequence. Figure 12 presents only a part of isoelectronic sequence, ranging from $Z = 50$ up to $Z = 72$ because the transition probabilities dependence is smooth for higher $Z$ values. Furthermore, only the strong transitions out of all possible ones from this level are given here. It is evident from

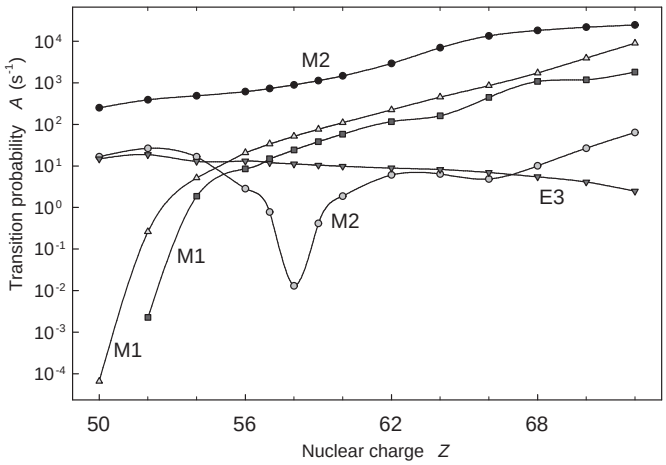


FIG. 12: Radiative transition probabilities originating from the level $4p^6 4d^5\,(^2I)4f\ ^3K_8$ along the molybdenum isoelectronic sequence. Two M1 transitions are presented by black squares and white triangles, two M2 transitions are presented by black and grey circles, E3 transitions are presented by grey triangles.

this figure, that one M2 transition is stronger than all other, but probability of another M2 transition has a sharp minimum near $Z = 58$. Whereas the probabilities of the M1 transitions evenly increase, this minimum of the latter M2 transition causes the dip in Fig. 11.

A radiative transition probability $A$ is proportional to a square of the transition operator matrix element. When this matrix element changes its sign, its value becomes very close to a zero at some particular value of $Z$. Consequently, the square of the radiative transition operator matrix element has a sharp minimum at this $Z$, causing such a peculiar shape of $A(Z)$. Very similar behavior of radiative transition parameters proportional to a square of matrix element one can see in [5].

### D. Rh-like ions

Figure 13 displays the energy level structure of investigated configurations for the rhodium-like ion with $Z = 56$. These ions are the most complex ones, with regard to metastable levels, when considering sequences with open $4d$ shell. There is a large number of excited levels having large $J$ values, whereas the ground configuration is $4p^6 4d^9\ ^2D$.

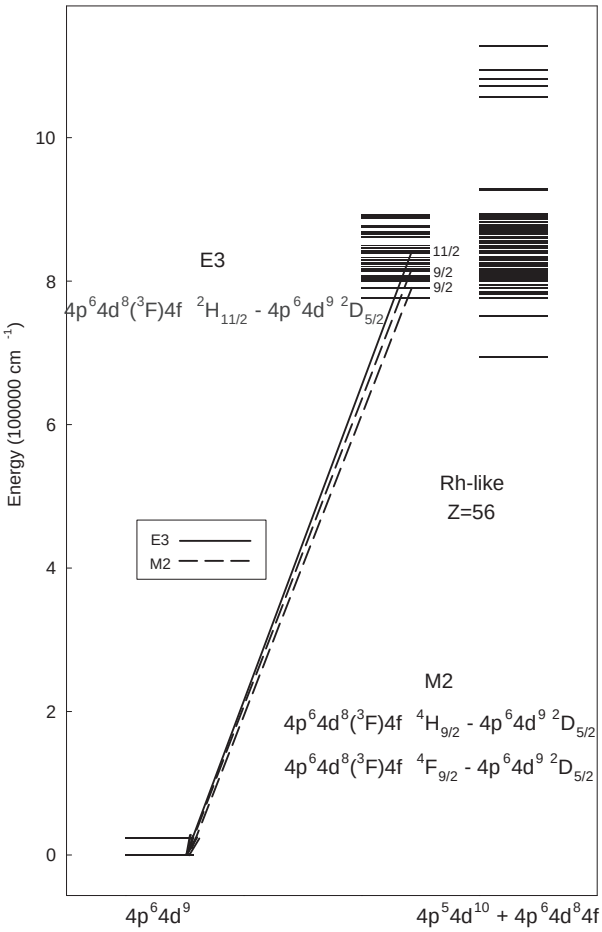


FIG. 13: Energy levels, E3 and M2 transitions for rhodium-like barium ion

So, only two levels, $J = 3/2$ and $J = 5/2$ make up the ground configuration. We have placed into a separate column all the levels with $J \geq 9/2$ from the configuration complex $4p^5 4d^{10} + 4p^6 4d^8 4f$. There are two levels with $J = 15/2$, five levels with $J = 13/2$, nine $J = 11/2$ levels and thirteen levels with $J = 9/2$. These 29 levels make up one third of complete set of excited odd-configuration levels. Not only E1, but also E3 and M2 transitions are forbidden from the levels with $J = 13/2$ and $J = 15/2$. Therefore these levels can decay only through transitions inside their configuration complex. The properties of these levels will be discussed further in this section. The significance of the transitions from three levels presented in Fig. 13 is studied further in this section where the radiative lifetimes $\tau_{\rm TOT}$ of these metastable and some other levels and their possible decay channels are discussed too.

Properties of the energy levels for the ions from the rhodium isoelectronic sequence are

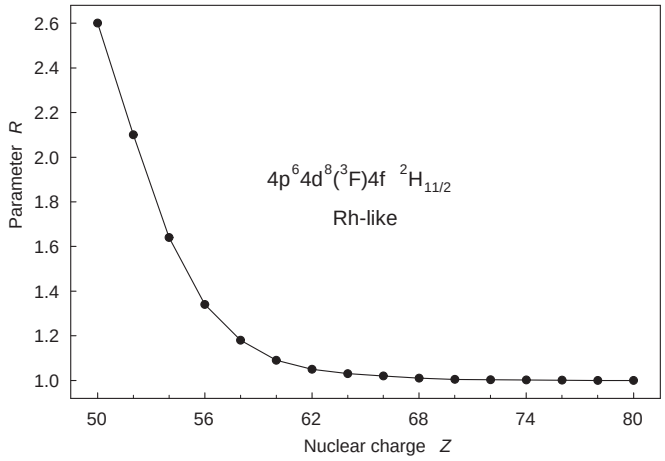


FIG. 14: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^8\,(^3F)4f\ ^2H_{11/2}$ of rhodium-like ions

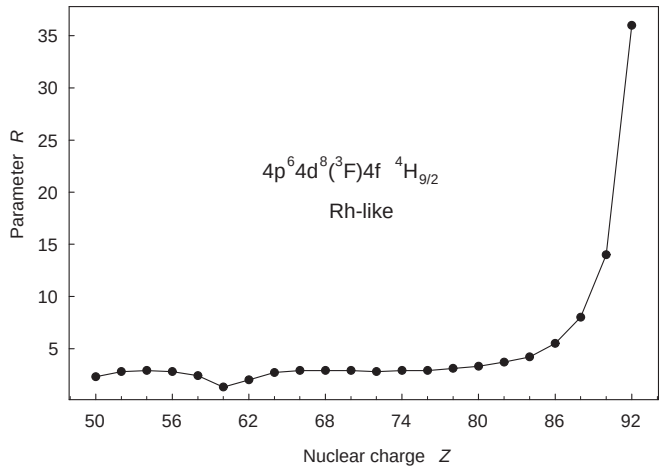


FIG. 15: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^8\,(^3F)4f\ ^4H_{9/2}$ of rhodium-like ions

presented in Figs. 14, 15, 16. Only the E3 transitions to the ground configuration are allowed from the level $4p^6 4d^8\,(^3F)4f\ ^2H_{11/2}$ (Fig. 14). The value of $R$ is comparatively small and it decreases rapidly. For the $4p^6 4d^8\,(^3F)4f\ ^4H_{9/2}$ level, the parameter $R$ increases sharply (see Fig. 15) only at the end of the sequence. Quite a large number of transitions contribute to the defined radiative lifetime $\tau_{\rm TOT}$ value of this level, such as strong M2 transition, two weak E3 transitions and several different M1 transitions. A sharp increase of $R$ is caused by the M2 transition probability being larger than that of the M1 transitions and increasing

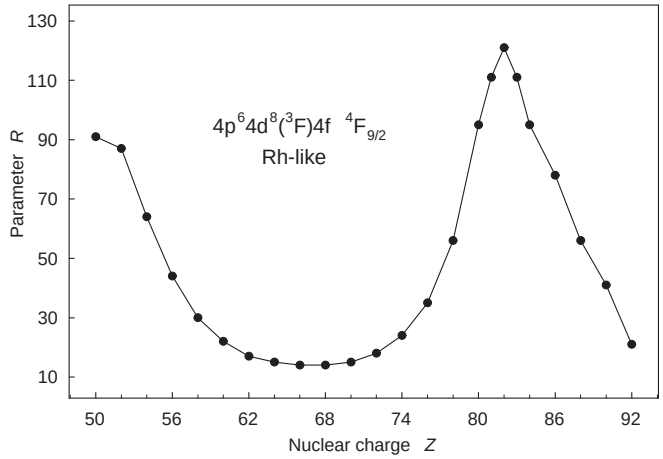


FIG. 16: Dependence of parameter $R$ on nuclear charge $Z$ for the level $4p^5 4d^8\,(^3F)4f\ ^4F_{9/2}$ of rhodium-like ions

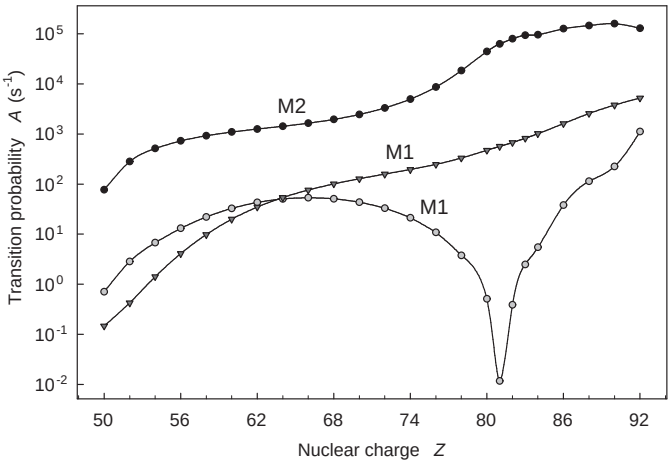


FIG. 17: Radiative transition probabilities originating from the level $4p^5 4d^8\,(^3F)4f\ ^4F_{9/2}$ along the rhodium isoelectronic sequence. Two M1 transitions are presented by grey circles and grey triangles, M2 transitions are presented by black circles

significantly faster. A small irregularity at $Z = 60$ appears due to a minimum of the dominant M2 transition probability value at these $Z$.

The most interesting behavior of the parameter $R$ can be seen in Fig. 16, presenting the level $4p^5 4d^8\,(^3F)4f\ ^4F_{9/2}$. To explain it, we present calculated radiative transition probabilities from this level in Fig. 17. In the first part of the isoelectronic sequence, $R$ demonstrates quite an usual behavior. Here the influence of the strongest M2 transition decreases,

although the probability values of that transition increase. Nevertheless, this increase is slower comparing to increase of the M1 transition probability values. When $Z$ is close to 70, the M1 transition probability values begin to decrease and they reach minimum at $Z = 81$. The decrease and subsequent increase is so sharp that we have performed calculations for several additional ions in the proximity of these $Z$ values. As it is mentioned in previous sections, such a dependence of the radiative transition probabilities is caused by the behavior of the corresponding radiative transition operator matrix element. For other levels with $J = 9/2$ and $J = 11/2$, the parameter $R$ values are small, because these levels are located comparatively high in energy spectra and have strong decay channels through M1 transitions.

As it was mentioned above, only the transitions inside excited configurations complex are allowed from levels with $J = 13/2$ and $J = 15/2$. In particular parts of the rhodium sequence, even M1 and E2 transitions are not possible for these metastable levels. This is caused by the location of these levels in energy spectra. The level with the highest $J$ value of $4p^6 4d^8\,(^3F)4f\ ^4I_{15/2}$ is too low to have any radiative transition when $50 \leq Z \leq 60$, because the lower energy levels only have $J < 9/2$. For the higher-$Z$ ions, the $4p^6 4d^8\,(^3F)4f\ ^2I_{13/2}$ level energy is lower than that of $^4I_{15/2}$, therefore rather weak M1 transition becomes possible. The determined radiative lifetime of this level $\tau = 2500$ s for $Z = 52$ and decreases to $\tau = 5.7 \cdot 10^4$ ns for $Z = 92$.

The level $4p^6 4d^8\,(^3F)4f\ ^2I_{13/2}$ does not have a radiative decay channel when $82 \leq Z \leq 92$. When $Z = 80$, only one level with $J$ value high enough, namely, $4p^6 4d^8\,(3F)4f\ ^2G_{9/2}$ lies below the level with $J = 13/2$. Therefore, the E2 radiative transition becomes possible, but the calculated radiative lifetime $\tau_{\mathrm{E2}} = 1.5 \cdot 10^9$ s. This equals to approximately 46 days. The low lying level $4p^6 4d^8\,(^3F)4f\ ^2G_{9/2}$ also has some peculiar properties. For $Z < 66$, it has only one radiative decay channel via the M2 transition. When $Z \geq 66$, a decay channel through the M1 radiative transition is open, but due to a negligible transition probability values, it becomes somehow significant only when $Z = 86$. That means that only the magnetic quadrupole transition is meaningful for this level along all isoelectronic sequence.

There is one more quite unusual level $4d^8\,(^3F)\,4f\ ^2S_{1/2}$ in this isoelectronic sequence. Formally, it does not satisfy our criteria, because the E1 transitions are allowed from it. Following comprehensive studies, we have noticed that, beside electric dipole transition to the $^2D_{3/2}$ level of the ground configuration, the M2 transition to the $^2D_{5/2}$ level becomes

significant starting from $Z = 78$. When $Z$ increases, the value of M2 transition probability becomes similar to that of E1 transition.

### E. Remaining $4d^N$ sequences

We must highlight that in current work we present only a small part of our results. We have investigated metastable levels in all isoelectronic sequences with $4d^N$ shell in the ground configuration. Magnetic dipole transitions are the most common decay channel for the levels of excited configurations. In previous sections we have described the cases when the E3 and M2 radiative transitions become significant decay channels. As a rule, when the M2 radiative transitions are possible, these are much stronger comparing to the E3 transitions originating from the same level. In some particular cases, the E3 transition probabilities can be of the same magnitude as those of the M2 transitions. Moreover, there are energy levels with the highest $J$ values, which can not decay by the M2 transitions.

Furthermore, the radiative decay is possible to other lower levels with a suitable $J$ value, not only to those given in our plots. It was established, that, for some levels, the M2 and E3 transitions are significant only in a narrow $Z$ range of their isoelectronic sequence. In current work, we present the most typical dependences, where the parameter $R$ decreases when nuclear number $Z$ increases. Other particular cases, when $R$ increases with $Z$ increase, are given here too along with some more peculiar behavior of the parameter $R$. All these statements are valid both for the sequences described in Sects. III A, III B, III C, III D and for remaining $4d^N$ sequences with $N = 1, 3, 5, 7, 8, 10$.

The ground configuration for Rb-like ions is $4p^6 4d\ ^2D$. The first two excited configurations do not have energy levels with significant M2 and E3 transitions. This is caused by the fact that levels with maximum $J$ values are located comparatively high, therefore M1 and E2 transitions to the lower levels of the same configurations are allowed. Only the level $4p^5 4d^2\ (^3F)\ ^4F_{9/2}$ has $R > 2$ at the beginning of the sequence, but it becomes $R < 1.1$, when $Z = 64$.

The ground configuration for yttrium-like ions is $4p^6 4d^3$. The M2 and E3 transitions do not play an important role here because the configuration $4p^6 4d^3$ has levels with large $J$ values ($J_{\max} = 11/2$). Only four excited levels have $J = 15/2$ and just two of them have the M2 and E3 transitions with significant probability values. For the level $4d^2\ (^3F) 4f\ ^4I_{15/2}$,

the $R > 12$ at the beginning of the isoelectronic sequence when $Z = 52$ but it decreases sharply when $Z$ increases. Such a behavior is caused by one M2 transition with transition probability value more than 100 times larger than the value of strongest M1 transition at the beginning of isoelectronic sequence. At the top end of the isoelectronic sequence, the M2 transition probability becomes similar to that of several M1 transitions which determine radiative lifetime of this level. Another level, namely $4p^5 4d^4\,(^3H)\;{}^4I_{15/2}$, has $R_{\max} = 4$ at the beginning of the isoelectronic sequence. For the higher-$Z$ ions, $R$ decreases rather slowly comparing to other levels. Such a behavior is caused by fact that decay of this level is going through many channels. There are three E3 transitions and one M2 transition to the ground configuration levels in addition to M1 and E2 transitions among the levels of this configuration. Moreover, this level is rather special one, because at higher $Z$ values, the E2 transitions become more important than M1 transitions for the calculated radiative lifetime $\tau_{\rm TOT}$, contrary to the most levels in this and other investigated isoelectronic sequences.

The ground configuration for niobium isoelectronic sequence is $4p^6 4d^5$. The excited configuration levels with $J = 19/2$ are located rather high in energy level spectra. Therefore, strong M1-type transitions are possible to lower-lying levels with $J = 17/2$. There are six levels with $J = 17/2$, but only two of them have their radiative lifetimes significantly affected by transitions of higher multipolar order. For the first of them, $4d^4\,(^3H)4f\;{}^4K_{17/2}$, the parameter $R$ gradually decreases from the top value of $R = 255$ with $Z$ increase. The decay of this level is going mainly through a strong M2 transition. Meanwhile, the parameter $R$ changes non-monotonously along the $Z$ and has a maximum value of $R = 39$ at $Z = 72$ for the level $4d^4\,(^3H)4f\;{}^2L_{17/2}$. Such a non-standard behavior can be explained in a similar way as it has been done for Mo-like ions (Fig. 11) and for Rh-like ions (Fig. 16).

The ground configuration for technetium-like ions is $4p^6 4d^7$. For the excited-configuration levels with $J = 17/2$, only the E3 transitions are allowed. Furthermore, these transitions are weak and have some significance only for the level $4d^6\,(^3H)4f\;{}^2L_{17/2}$ at the lower end of this isoelectronic sequence. The parameter $R = 4$ at $Z = 52$, and it decreases sharply for higher $Z$ values. There are fifteen energy levels with $J = 15/2$, but only two of them are significantly affected by the M2 and E3 radiative transitions. For other levels, the influence of these transitions is insignificant even at the low end of the isoelectronic sequence.

In the case of $4d^6\,(^3H)4f\;{}^2K_{15/2}$ level, inclusion of the M2 decay channel changes the value of radiative lifetime $\tau_{\rm TOT}$ more than 4000 times at the beginning of the isoelectronic

sequence. It decreases fast down to $R = 7$ at $Z = 76$. On the other hand, the M2 transition is not so important for the level $4d^6\,(^3H)4f\ ^4I_{15/2}$. The parameter $R$ is approximately equal to 40 at the beginning of the sequence. It decreases rather slowly, and $R = 2$ when $Z = 76$.

The ground configuration for Ru-like ions is $4p^6 4d^8$. The total angular momentum $J$ maximum values can be 9 and 8 for the excited configurations $4p^6 4d^7 4f$ and $4p^5 4d^8$. The E3 and M2 transitions from these levels are forbidden. They have one or more M1 transitions to lower levels with $J = 8$ and $J = 7$ from the same configuration complex.

An interesting case constitutes the level $4d^7\,(^4F)4f\ ^3I_7$. The main decay channels for this level are the radiative E3 and M1 transitions. The parameter $R$ increases from the beginning of the isoelectronic sequence (like in Fig. 4) and reaches maximum value of $R = 30$ at $Z = 92$.

There are nineteen $J = 6$ levels, most of them having radiative lifetimes $\tau_{\rm TOT}$ significantly affected by transitions of higher multipole orders. This is especially noticeable at the beginning of isoelectronic sequence where the parameter $R$ values are close to 10. Very high $R$ values were determined for three lowest energy levels with $J = 6$. The level $4d^7\,(^4F)4f\ ^5G_6$ is very special one because of possible M2 transition being virtually the only available decay channel (except for very weak E2 transition), therefore $R > 900,000$ when $Z = 52$. The value of $R$ becomes smaller than 1000 only starting with $Z > 80$ ions. Only in the case, when some level with $J = 5$ has an energy lower than this $4d^7\,(^4F)4f\ ^5G_6$ level, the decay through the M1 transition becomes possible. For the levels $4d^7\,(^4F)4f\ ^3H_6$ and $4d^7\,(^2G)4f\ ^1I_6$, the $R$ values reach 30 and 40, correspondingly, at the beginning of this isoelectronic sequence, but they decrease sharply for higher-$Z$ ions.

The ground configuration of palladium isoelectronic sequence is $4p^6 4d^{10}$. Therefore, only one excited configuration $4p^6 4d^9 4f$ can be considered. Only three levels from this excited configuration can decay through E1 transitions, because the ground configuration has the only $^1S_0$ level. Two levels, $4d^9\,(^2D)4f\ ^3P_0$ and $4d^9\,(^2D)4f\ ^3H_0$, have no radiative decay channels at all. In general, the radiative lifetimes of this configuration levels are determined by M1 transitions. Certainly, E2 transitions are allowed, too, but their transition probabilities are significantly smaller than those of M1 transitions. However, there are levels where transitions of higher multipole order play an important role. Four levels with $J = 2$ can decay not only through M1 transitions but also via M2 transitions. This channel is extremely important for the levels $4d^9\,(^2D)4f\ ^3F_2$ and $4d^9\,(^2D)4f\ ^3P_2$. The parameter $R$ decreases from 800 to 10 when $Z$ changes from 52 to 92, i.e. over all range of examined

ions, for the $^3F_2$ level and from 1400 to 5 for the $^3P_2$ level. For other two levels with $J = 2$, the influence of M2 transitions on total radiative lifetime $\tau_{\rm TOT}$ is not so very large but still rather noticeable. The M1 and E3 transitions are allowed from the levels with $J = 3$. The E3 transitions are not significant comparing to M2 transitions. Therefore $R$ values are close to 1. Only the level $4d^9\,(^2D)4f\ ^1F_3$ makes some exception having $R$ close to 4 at $Z = 52$. Nevertheless, it decreases fast and reaches 1.4 at $Z = 60$.

## IV. CONCLUSIONS

The magnetic quadrupole and electric octupole transitions from some levels of the first excited configurations $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$ to the ground configuration $4p^6 4d^N$ can play very significant role for their radiative lifetimes. These levels have large total angular momentum $J$ values, therefore the radiative decay to the ground configuration via electric dipole transitions is not allowed. Consequently, these levels are metastable ones, therefore one assumes that the M1 and E2 transitions shall become the main decay channels and determine the lifetimes of such excited levels. Nevertheless, since such levels are located in the lower part of their own configuration complex energy spectra, possible transitions (M1 and E2) among the excited configuration $4p^5 4d^{N+1}$ and $4p^6 4d^{N-1} 4f$ levels are severally restricted. Our investigation has demonstrated that radiative transitions of higher multipole order, such as M2 and E3, become significant.

Inclusion of additional decay through radiative M2 and E3 transition channels decreases values of determined radiative lifetimes $\tau_{\rm TOT}$. The main impact on calculated radiative lifetimes $\tau_{\rm TOT}$ originates from M2 transitions if these are allowed by $J$ selection rules, whereas E3 transition probability values are smaller. The E3 transitions become significant when $J$ acquires maximum values and therefore the M2 transitions are forbidden.

The influence of E3 and M2 transitions can both to decrease and to increase along the isoelectronic sequence. This originates both from the variation of transition probability values and from the location changes of investigated levels in respect to other levels of the same configuration and variation of corresponding M1 and E2 transition probabilities.

Most of investigated levels belong to configurations $4p^6 4d^{N-1} 4f$. It is simple to explain that such an excitation always allows to get large values of the final $LS$ momenta and the total momentum $J$. For the excitation of the $4p$ electrons to the states with $4d$ electrons,

large total angular momenta $J$ appear only in the case when a number of electrons in $4d$ shell is small. When $N$ is large ($N = 9, 10$), there exists levels with forbidden both E1, E2, E3 and M1,M2 transitions.

Performed investigation has demonstrated that it is difficult to predict the significance of the E3 and M2 transitions without performing detailed calculations. This is consequence of the fact that the values of different radiative transition probabilities can change dramatically along the isoelectronic sequences due to changes of the eigenfunctions and transition energies.

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